Direction-Cosine Attitude-Control Logic for Spin-Stabilized Axisymmetric Spacecraft

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The developments of this study yield a control logic for the active attitude control of a spin-stabilized axisymmetric spacecraft. The derived control logic makes use of direction cosines for attitude definition, and is not restricted by either small angle assumptions or by the kinematic singularities associated with Euler angles. The active torqueing capability is provided by means of a reaction-jet system, and the control logic is simplified by assuming that control torques may be applied impulsively. The control logic formulated is optimal in the sense that each control impulse is delivered in such a manner as to cause a maximum deduction in "system error."

Nomenclature

I_i = moment of inertia for the x_i principal axis
M_i = external torque acting on the x_i axis
ω_i = components of the angular velocity vector in the x_i system
Ω = $\omega_3(t)$ = spin velocity
$u(t)$ = control magnitude defined by Eqs. (1)
$\eta(t)$ = control phase defined by Eqs. (1)
I = $I_1 = I_2$
a = $(I_3 - I)/I$
$[A]$ = direction cosine matrix
γ_i = angle between the x_i axis and the X_3 inertial axis
a_{i3} = $\cos\gamma_i$
ω = $\omega_1 + i\omega_2$
α = $a_{13} + ia_{23}$
θ = phase of ω
ϕ = phase of α
$|J^i|$ = magnitude of control impulse i
η^i = phase of control impulse i
b = I_3/I = inertial parameter
μ = $b\Omega$
$|H|$ = magnitude of rigid-body angular momentum vector
β = $|H|/I$ = precession rate
δ = $\tan^{-1}(I|\omega^0|/I_3\Omega)$ = coning angle
λ = angle between the angular momentum vector H and the inertial X_3 axis
ζ^0 = spherical angle defined in Eqs. (25), and illustrated in Fig. 2
E = error function defined in Eq. (29)
G = optimization criterion to be maximized defined in Eq. (30)
ξ = the angle between the $X_3 - x_1$ and $X_3 - x_2$ planes
ψ = the angle between the $X_3 - x_1$ and $X_3 - H$ planes
$\bar{\eta}^1$ = $\eta^1 - \pi$

Subscripts

1,2,3 = refer to the x_1, x_2, x_3 axes

Superscripts

0 = refers to an initial time $t = 0$
1 = refers to either the time at which the control impulse $J^1\delta(t - t^1)$ is applied, or to the state of a variable or parameter immediately following the impulse

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Introduction

THE basic objective of the analysis that follows is the synthesis of an active feedback attitude-control logic for spin-stabilized axisymmetric spacecraft. Control torques are to be generated by means of a reaction-jet system, and the proposed control logic is simplified by assuming that control torques may be applied impulsively. Pulse-width modulation is suggested for generating the derived control impulses.

Windeknecht[1] was the first of several authors[2,3] to suggest impulsive control logic for passively damped spacecraft. Since passive damping systems are only effective if the spin-axis moment of inertia is greater than either transverse-axis moment of inertia,[4] the control logic suggested in Refs. 1–3 is not workable for many spacecraft configurations. In contrast to these systems, Porcelli and Connolly[5] have suggested a control logic for the active control of slender (i.e., "pencil-shaped") spacecraft and dual-spin spacecraft.[6] Childs[7] has recently suggested a control logic which is applicable for axisymmetric spacecraft of arbitrary inertial proportions; i.e., it could be used to control a "disk-shaped" body or a "pencil-shaped" body. It is not appropriate, however, for a spherical body.

All of the previously cited control approaches employ Euler angles as kinematic variables and are restricted in application to situations which do not require "large" angular reorientations. The control logic suggested by Porcelli and Connolly and by Childs are based on linearized Euler angle models which become questionable for angular reorientations in excess of 15° (approximately). Although these control approaches could be extended by changes in reference, a more fundamental approach consists of employing a kinematical representation which does not have the inherent kinematic singularity associated with Euler angles. The components of the direction cosine matrix[8,9] are such a representation, and the analysis of this study makes use of them in a development similar to that of Childs.[7]

Control Model

The basic attitude-control requirement for spin-stabilized spacecraft is that the spin axis be placed and maintained within some small defined neighborhood of a prescribed orientation. The physical variables that must be controlled are angular rates and angular displacements. The angular rates may be defined by Euler's equations of motion for a rigid body which, for an axisymmetric body, are stated as

$$\begin{aligned} \dot{\omega}_1 + a\Omega\omega_2 &= M_1 I = u(t)\cos\eta(t) \\ \dot{\omega}_2 - a\Omega\omega_1 &= M_2/I = u(t)\sin\eta(t) \\ \omega_3(t) &= \omega_3(0) = \Omega \end{aligned} \tag{1}$$

where

$$a = (I_3 - I)/I \tag{2}$$

and the subscripts 1, 2, 3 identify body-fixed x_1, x_2, x_3 principal axes with the x_3 axis the axis of symmetry. The origin of the x_1, x_2, x_3 system coincides with the mass center of the rigid body. The variables ω_1, ω_2, ω_3 and M_1, M_2 are, respectively, the components of the angular velocity vector of the body and the external torque vector. The parameters I and I_3 are, respectively, the transverse ($I_1 = I_2 = I$) and spin-axis moment of inertias. The form of Eqs. (1) implies that control is to be supplied by a gimballed torqueing system, i.e., that $\eta(t)$ is an unbounded control variable.

The angular orientation of the body-fixed x_i system relative to an inertial X_i system may be defined by the direction cosine matrix $[A]$. If the components of the arbitrary vector v in the x_i and X_i systems are denoted, respectively, by (v_i) and $(v)_I$, the direction cosine matrix satisfies

$$(v)_i = [A](v)_I; \quad (v)_I = [A]^T(v)_i \tag{3}$$

where T denotes the matrix transpose operation. Further, the matrix $[A]$ is related[8,9] to the components of the angular velocity vector cited in Eqs. (1) by

$$[\dot{A}] = -[(\omega)][A] \tag{4}$$

where

$$[(\omega)] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{5}$$

Since only the spin-axis orientation is significant, one extracts from Eq. (4)

$$\begin{aligned} \dot{a}_{13} &= \Omega a_{23} - \omega_2 a_{33} \\ \dot{a}_{23} &= -\Omega a_{13} + \omega_1 a_{33} \\ \dot{a}_{33} &= \omega_2 a_{13} - \omega_1 a_{23} \end{aligned} \tag{6}$$

These variables are not independent, since they satisfy the kinematic constraint

$$a_{13}^2 + a_{23}^2 + a_{33}^2 = 1 \tag{7}$$

Equations (1) and (6) constitute the system of governing equations. The kinematic definition given in Eqs. (6) is not restricted by either small angle approximations or by kinematic singularities.

The control logic derived in this study is based on the assumption that the effect of a "short-duration" firing of the gimballed reaction jet can be adequately approximated by an impulse. Since a reaction jet is essentially an "on-off" device, i.e.,

$$u = U > 0, \text{ or } u = 0$$

the magnitude of a control impulse is approximately defined by

$$J = UI\Delta t$$

where Δt is the firing duration. By varying Δt one obtains impulses of varying magnitude. This approach is commonly referred to as pulse-width modulation.

To determine the effect of a control impulse, Eqs. (1) and (6) are restated as

$$\dot{\omega} = ia\Omega\omega + \hat{u}(t) \tag{8}$$

and

$$\begin{aligned} \dot{\alpha} &= ia_{33}\omega - i\Omega\alpha \\ \dot{a}_{33} &= \omega_2 a_{13} - \omega_1 a_{23} \end{aligned} \tag{9}$$

where

$$\begin{gathered} \omega = \omega_1 + i\omega_2 = |\omega|e^{i\theta}; \quad \alpha = a_{13} + ia_{23} = |\alpha|e^{i\phi} \\ \hat{u}(t) = u(t)e^{i\eta(t)} \end{gathered} \tag{10}$$

The solution to Eq. (8) for the control impulse

$$u(t) = (J^1/I)\delta(t - t^1) = (|J^1|/I)e^{i\eta_1}\delta(t - t^1) \tag{11}$$

can be expressed as

$$\begin{aligned} \omega(t) &= \omega^0 e^{ia\Omega t} \qquad 0 \le t < t^1 \\ \omega(\tau) &= \omega^1 e^{ia\Omega\tau} \qquad 0 \le \tau \end{aligned} \tag{12}$$

where

$$\begin{gathered} \tau = t - t^1, \ \omega^0 = \omega(0) = |\omega^0|e^{i\theta^0} \\ \omega^1 = \omega(t^1) = \omega^0 e^{ia\Omega t^1} + (J^1/I) \end{gathered} \tag{13}$$

From Eq. (12), Eq. (9) is reduced to a system of linear time-varying differential equations. The transformation

$$\alpha = ze^{i(a\Omega t + \theta^0)} = (z_1 + iz_2)e^{i(a\Omega t + \theta^0)} \tag{14}$$

further reduces Eq. (9) to the linear time-invariant system

$$\begin{aligned} \dot{z} + i\mu z &= i|\omega^0|a_{33} \\ \dot{a}_{33} &= -|\omega^0|z_2 \end{aligned} \tag{15}$$

where

$$b = a + 1 = I_3/I, \ \mu = b\Omega \tag{16}$$

The solution of Eq. (15) is readily obtained, and from Eq. (14) yields for $t < t^1$

$$\begin{aligned} \alpha(t) = {} & \alpha^0 e^{ia\Omega t} - i|\alpha^0||\omega^0|^2(\beta^0)^{-2}\sin(\phi^0 - \theta^0) \times \\ & (1 - \cos\beta^0 t)\exp[i(a\Omega t + \theta^0)] + (a_{33}{}^0\omega^0 - \mu\alpha^0) \times \\ & (\beta^0)^{-2}[\mu(1 - \cos\beta^0 t) + i\beta^0 \sin\beta^0 t]e^{ia\Omega t} \end{aligned} \tag{17}$$

and

$$\begin{aligned} a_{33}(t) = {} & a_{33}{}^0(\mu^2 + |\omega^0|^2\cos\beta^0 t)/\beta^0 + \\ & \mu|\alpha^0|\omega^0|\cos(\phi^0 - \theta^0)(1 - \cos\beta^0 t)/\beta^0 - \\ & |\alpha^0||\omega^0|\sin(\phi^0 - \theta^0)\sin\beta^0 t/\beta^0 \end{aligned} \tag{18}$$

where

$$\beta^0 = (\mu^2 + |\omega^0|^2)^{1/2} = [(I_3\Omega)^2 + (I|\omega^0|)^2]^{1/2}/I \tag{19}$$

Hence,

$$\beta^0 = |H^0|/I \tag{20}$$

where H^0 is the initial angular-momentum vector of the rigid body. The solution for $t \ge t^1$ (or $\tau \ge 0$) is obtained from Eqs. (17–20) by replacing the index 0 by 1 and substituting τ for t.

Although the solution developed above is formally correct, it is physically unmotivating, and a more meaningful solution format is obtained via the following definitions.

$$\sin\delta^0 = I|\omega^0|/|H^0| = |\omega^0|/\beta^0$$
$$\cos\delta^0 = I_3\Omega/|H^0| = \mu/\beta^0 \tag{21}$$

$$a_{33} = \cos\gamma_3,\ |\alpha| = \sin\gamma_3 \tag{22}$$

$$H_{X3}{}^0 = |H^0| \cos\lambda^0 \tag{23}$$

The angles defined above are illustrated in Fig. 1, and one observes that λ^0 is the angle between the angular-momentum vector H^0 and the inertial X_3 axis, while δ^0 is the angle between the angular-momentum vector and the spin axis. The angle γ_3 obviously lies between the x_3 axis and the X_3 axis.

Of these three angles, λ^0 and δ^0 are piecewise-constant functions of time (stepping discontinuously when an impulse is applied), while γ_3 is a continuous function of time. From the last of Eqs. (3) and (23), one obtains

$$\cos\lambda^0 = \cos\gamma_3{}^0 \cos\delta^0 + \sin\gamma_3{}^0 \sin\delta^0 \cos(\phi^0 - \theta^0) \tag{24}$$

which yields, in conjunction with Eqs. (21) and (22),

$$a_{33}(t) = \cos\delta^0 \cos\lambda^0 + (\cos\gamma_3{}^0 - \cos\delta^0 \cos\lambda^0) \cos\beta^0 t - \sin\gamma_3{}^0 \sin\delta^0 \sin(\phi^0 - \theta^0) \sin\beta^0 t$$

for Eq. (18). This result is further simplified by the substitution

$$\sin\zeta^0 = \sin\gamma_3{}^0 \sin(\phi^0 - \theta^0)/\sin\lambda^0$$
$$\cos\zeta^0 = (\cos\gamma_3{}^0 - \cos\delta^0 \cos\lambda^0)/\sin\delta^0 \sin\lambda^0 \tag{25}$$

which yields

$$a_{33}(t) = \cos\delta^0 \cos\lambda^0 + \sin\delta^0 \sin\lambda^0 \cos(\zeta^0 + \beta^0 t) \tag{26}$$

The planar representation of Fig. 2 illustrates the spherical trigonometry involved in this torque-free rigid-body solution with the spin axis precessing about the angular momentum vector at the rate β^0. Eqs. (24) and (26) are simply an expression of the law of cosines for arcs from spherical trigonometry.

Control Synthesis

By a suitable definition of the inertial X_i system, the general attitude-control objective can be interpreted as the requirement that the x_3 axis (spin axis) be placed and maintained in coincidence with the inertial X_3 axis. In terms of the state variables, the desired terminal state is then defined by

$$\omega_1(T) = \omega_2(T) = a_{13}(T) = a_{23}(T) = 0$$
$$a_{33}(T) = 1 \Longrightarrow \gamma_3(T) = 0 \tag{27}$$

where T is the first time for which Eqs. (27) are satisfied, and is unspecified.

From Eq. (26), the solution for a_{33} following application of a control impulse is given by

$$a_{33}(\tau) = \cos\delta^1 \cos\lambda^1 + \sin\delta^1 \sin\lambda^1 \cos(\zeta^1 + \beta^1\tau) \tag{28}$$

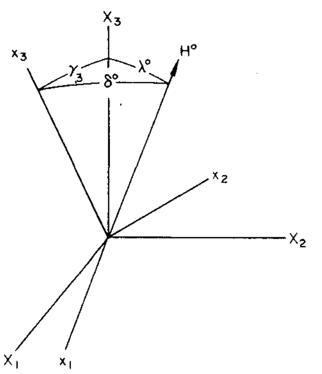

Fig. 1 Illustrative definition of the angles λ^0, δ^0, γ_3.

A given control impulse will be defined as optimal if it minimizes the system error that follows the impulse. In view of the desired terminal conditions cited in (27), system error is defined as the maximum value of the spin axis angle γ_3, i.e.,

$$E = \max\gamma_3(t) \tag{29}$$

Hence, the initial system error is defined by $E^0 = \lambda^0 + \delta^0$. Equivalently, a control impulse is optimal if it maximizes the minimum value of $a_{33}(\tau)$ which occurs following a control impulse. From Eq. (28) the quantity to be maximized is

$$G(|J^1|,\eta^1,t^1) = \cos(\lambda^1 + \delta^1) \tag{30}$$

The necessary conditions of optimality, $\partial G/\partial|J^1| = \partial G/\partial t^1 = \partial G/\partial\eta^1 = 0$, are satisfied by

$$\sin(\zeta^0 + \beta^0 t^1) = 0, \quad \cos(\zeta^0 + \beta^0 t^1) = 1 \tag{31}$$

$$\sin(a\Omega t^1 + \theta^0 - \eta^1) = 0,\ \cos(a\Omega t^1 + \theta^0 - \eta^1) = -1 \tag{32}$$

Although the formal confirmation of these results is lengthy, their kinematic significance is easily appreciated from Fig. 3. Eqs. (31) imply that the optimal firing time t^1 occurs when $a_{33}(t)$ is a maximum (or $\gamma_3(t)$ is a minimum), whereas Eqs. (32) imply that the "new" angular-momentum vector H^1 is to lie in the plane defined by the initial angular-momentum vector H^0 and the inertial X_3 axis. This latter statement can be better appreciated by noting from Eqs. (13) that

$$|\omega^1|^2 = |\omega^0|^2 + 2|\omega^0||J^1|/I \cos(a\Omega t^1 + \theta^0 - \eta^1) + (|J^1|/I)^2$$

If in addition to Eqs. (32), one also has

$$|J^1| = I|\omega^0|$$

then $|\omega^1| = 0$, and the resultant angular-momentum vector H^1 would lie in the $H^0 - X_3$ plane. The firing time t^1 and impulse phase η^1 defined, respectively, by Eqs. (31) and (32) affect this desired result irrespective of the impulse magnitude $|J^1|$. As a consequence, one obtains

$$\lambda^1 + \delta^1 = \gamma_3{}^1 = \lambda^0 - \delta^0 \tag{33}$$

Note was made previously that the optimum firing time defined in Eqs. (31) occurs when $\lambda_3(t)$ is a minimum, which means that the optimum firing time can be determined from sensor measurements of $\lambda_3(t)$. From Eqs. (32) it then becomes necessary to measure the angular velocity phase $\theta(t) = \theta^0 + a\Omega t$ to determine the optimum firing angle $\eta^1 = \theta^0 + a\Omega t + \pi$. Since drift-free continuous measurement of the angular-velocity vector can be expeditiously accomplished by means of accelerometers,[10] the sensor requirements for Eqs. (31) and (32) are comparatively modest.

It is also possible to realize the desired control logic using only angular measurements, i.e., one need not measure the angular velocity phase $\theta(t)$. This can be accomplished from the kinematic interpretation given above and in Fig. 3 for Eqs. (31) and (32). The following additional angle definitions are required. The angles between the X_3 and x_1 axes and between the X_3 and x_2 axes are denoted, respectively, as

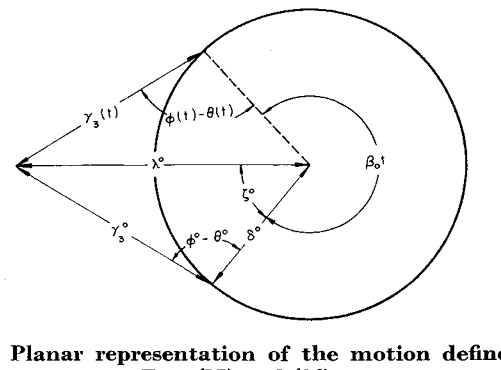

Fig. 2 Planar representation of the motion defined by Eqs. (17) and (26).

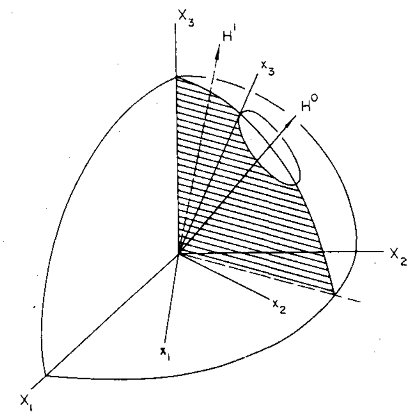

Fig. 3 Kinematic interpretation of Eqs. (31) and (32).

γ_1 and γ_2; hence, $a_{13} = \cos\gamma_1$, $a_{23} = \cos\gamma_2$. The angle between the $X_3 - x_1$ and $X_3 - x_2$ planes is denoted as ξ, while the angle between the $X_3 - x_1$ and $X_3 - H^0$ planes is ψ. Finally, one denotes $\bar{\eta}^1 = \eta^1 - \pi$. With these definitions one obtains from Fig. 4 via spherical trigonometry (the law of cosines for arcs)

$$\cos\bar{\eta}^1 = -\cos\gamma_1 \sin\gamma_3 + \sin\gamma_1 \cos\gamma_3 \cos\psi$$
$$\sin\bar{\eta}^1 = -\cos\gamma_2 \sin\gamma_3 + \sin\gamma_2 \cos\gamma_3 \cos(\xi - \psi) \tag{34}$$

and

$$0 = \cos\gamma_1 \cos\gamma_3 + \sin\gamma_1 \sin\gamma_3 \cos\psi$$
$$0 = \cos\gamma_2 \cos\gamma_3 + \sin\gamma_2 \sin\gamma_3 \cos(\xi - \psi) \tag{35}$$

Hence, from Eqs. (34) and (35)

$$\cos\bar{\eta}^1 = -\cos\gamma_1/\sin\gamma_3, \quad \sin\bar{\eta}^1 = -\cos\gamma_2/\sin\gamma_3 \tag{36}$$

and from Eqs. (10) and (22)

$$\cos\eta^1 = \cos\phi^1, \quad \sin\eta^1 = \sin\phi^1 \tag{37}$$

Eq. (37) may then be used in place of Eqs. (32) to define η^1.

The necessary conditions of optimality ($\partial G/\partial|J^1| = \partial G/\partial t^1 = \partial G/\partial\eta^1 = 0$) do not yield a unique value for $|J^1|$. A graphical illustration of this statement is provided in Fig. 5 where one notes (for $\lambda^0 > \delta^0$) that any $|J^1|$ in the range

$$I|\omega^0| \leq |J^1| \leq I|\omega^0| + I_3\Omega \tan(\lambda^0 - \delta^0) \tag{38}$$

yields the same optimal performance index, namely,

$$G^1|_{\text{opt}} = \cos(\lambda^0 - \delta^0) \Longrightarrow E^1 = \lambda^0 - \delta^0 \tag{39}$$

Similarly, one finds for the $\lambda^0 < \delta^0$ class of initial conditions that

$$I|\omega^0| - I_3\Omega \tan(\delta^0 - \lambda^0) \leq |J^1| \leq I|\omega^0| \tag{40}$$

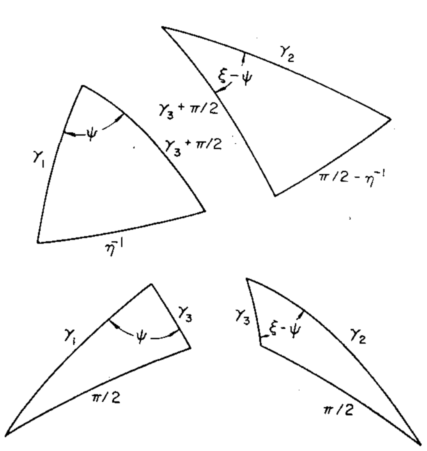

Fig. 4 Spherical trigonometry definitions.

and

$$G^1|_{\text{opt}} = \cos(\delta^0 - \lambda^0) \Longrightarrow E^1 = \delta^0 - \lambda^0 \tag{41}$$

In review then, the general solution for the optimal firing time t^1 and impulse angle η^1 are given, respectively, by Eqs. (31) and either Eqs. (32) or (37). The optimum impulse magnitude is defined by expression (38) if $\lambda^0 > \delta^0$ and by expression (40) if $\lambda^0 < \delta^0$. There are two special cases not covered by this general solution, namely, $\lambda^0 = 0$ and $|\omega^0| = 0$. In both cases, t^1 can be chosen arbitrarily, since $\gamma_3(t)$ is constant. In the $|\omega^0| = 0$ case, Eqs. (37) are used to define η^1 and $|J^1|$ is required to satisfy expression (38), whereas if $\lambda^0 = 0$, η^1 is defined by either Eqs. (32) or (37), and $|J^1|$ must satisfy expression (40). In both of these cases, the optimal control impulse does not reduce system error; however, any other nonzero control impulse increases system error. The solutions provided secure a global minimum for the error function defined in Eq. (29). Although a mathematical verification of this statement is both lengthy and tedious, its validity is established by Figs. 3 and 5.

A single control impulse, even if unrestrained in magnitude, can not in general achieve the desired terminal conditions given in Eqs. (27). Since control impulses will in fact be magnitude limited, a sequence of best impulses, $\Sigma J^i\delta(t - t^i)$, will be required to drive the system error into an acceptable neighborhood of the point defined by Eqs. (27). The constraint cited in expression (38) for optimal impulse magnitude (for $\lambda^0 > \delta^0$) makes no provision for physical limitations on impulse magnitudes $|J^i|$, although they are in fact bounded both above and below as follows

$$|J|_{\min} \leq |J^i| \leq |J|_{\max} \tag{42}$$

The lower bound arises because of physical limitations while the upper bound is necessary to insure that the "impulsive character" of control torques is preserved. With this in mind, two nonoptimum possibilities become evident in expression (38), namely,

$$|J|_{\max} < I|\omega^0| \tag{43}$$

and

$$|J|_{\min} > I|\omega^0| + I_3\Omega \tan(\lambda^0 - \delta^0) \tag{44}$$

Condition (43) would be likely to arise in the acquisition phase of control when $|J|_{\max}$ is too small to achieve the desired optimum given in Eq. (39), whereas condition (44) would be encountered near limit-cycle operation with $|J|_{\min}$ exceeding the allowable desired maximum. From Fig. 5, however, one would conclude that a control impulse will always reduce system error providing

$$|J|_{\min} < I|\omega^0| + I_3\Omega \tan\lambda^0 \tag{45}$$

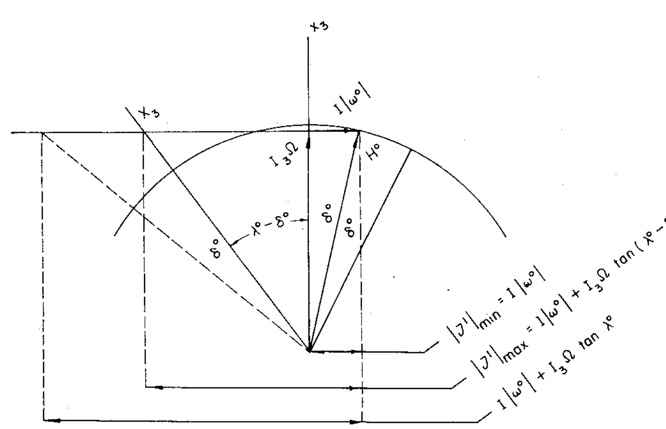

Fig. 5 Range of optimal and effective control-impulse magnitudes for $\lambda^0 > \delta^0$.

If control is applied when $|J|_{min} > I|\omega^0| + I_3\Omega \tan\lambda^0$, system error [as defined by max $\gamma(t)$] will be increased. This result also holds for $\lambda^0 < \delta^0$ initial conditions. Expression (45) effectively defines the idealized limit-cycle accuracy of the control logic in terms of $|J|_{min}$.

Summary and Conclusions

A control logic is developed herein which allows one to compute a sequence of optimum impulses that force the solution of Eqs. (8) and (9) into a predictable [expression (45)] idealized neighborhood of the point Eqs. (27). The control logic is not restricted by either small-angle approximations or by the kinematic singularities customarily associated with Euler angles. The control logic may be implemented by measurement of either the pointing error angle and the angular velocity vector[10] or by measurement of three angles (or three direction cosines).

The developments of this study are predicated on a gimballed torqueing system, and the resultant simplicity of the control logic is, to a large extent, purchased at the expense of mechanical complexity. There is no reason, however, that the analytical approach employed in this study could not be applied to a simpler mechanical system. For example, with a single-axis body-fixed torqueing system one would obtain for Eq. (1)

$$\dot{\omega}_1 + a\Omega\omega_2 = M_1/I = u_1,\ \dot{\omega}_2 - a\Omega\omega_1 = 0 \qquad (46)$$

where the control element u_1 is bounded by

$$u_1 = U_1, \text{ or } u = 0 \qquad (47)$$

Initial results with this model suggests that a "best-impulse" control logic can be formulated which will in general reduce the system error to a predictable and acceptable level. A control system using one body-fixed control torque is not as flexible as the gimballed control logic, and becomes inefficient for some inertia ratios. Use of one body-fixed jet does not, however, increase the sensor requirements. A detailed report on a single-axis control logic is in preparation.

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